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Bhaskaracharya: A Great Teacher

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Introduction

Let me first thank the organisers of the conference for giving me the opportunity to address this august gathering of highly experienced mathematics teachers. I consider this an honour to be asked to give Professor R. C. Gupta Memorial Lecture at this 49th Annual Convention of the Association of Mathematics Teachers of India (AMTI). Professor Gupta, as we all know, has made a name in the field of history of Mathematics. His understanding of the development of mathematics as a discipline is so deep that I hardly dare to enter into his realm. Instead, I will focus my attention on history through the eyes of a mathematics educator. I feel, I can play this role comfortably as I have spent major part of my life in developing Remedial Instructional Strategies in mathematics to teach school mathematics to socially disadvantaged students in India.

As you all know, mathematics is the most disliked subject in school curriculum. It is more so among the first generation learners. This is a group of students for whom there is to tradition of education at home. With a view to create interest among these students towards school mathematics I have been telling them stories based on historical anecdotes. An article based on this topic has already been published in one of the issues of Mathematics Teacher published by the Association of Mathematics Teachers of India. As I went on searching for more such anecdotes I came across the writings of some of the great Indian mathematicians. They were not only great as mathematicians but also were great as a teacher. A prominent example in this case is that of Bhaskaracharya, popularly known as Bhaskara II. Today, I would like to discuss how great he was as a teacher and how relevant his pedagogy is in the present context.

Bhaskaracharya

As per the historical records Bhaskaracharya was born at a place called Bijjal Bid close to Sahyadri ranges. Researchers believe that this place was close to Chalisgaon in the state of Maharashtra. A stone carving found near Patnadevi describes how knowledgeable he was. It states that Bhaskaracharya had studied grammar, natyshashtra and medicine apart from mathematics. The carving also mentions that he wrote Siddhanta Shiromani at the age of 36. The recorded year of his birth is shalivahan shaka 1036. Since there is a difference of 78 years

between Shalivahan calendar and Gregorian calendar the birth year of Bhaskaracharya as per Christian era comes to be 1114. Thus, 2014 happens to be the 900th birth anniversary of Bhaskaracharya. It is, therefore, appropriate to look at the pedagogy advocated by him through his writings at this stage. Bhaskaracharya was a prolific writer. He has written so much and I confess that my life is too short to read all his writings. Instead, I will concentrate my attention on his famous book “Lilavati”.

Lilavati

Lilavati, sometimes also called as patiganit, mainly discusses concepts in arithmetic. In addition, there are elementary concepts taken from Algebra and Geometry also. Here is short list of topics.

Arithmetic: Operations on numbers, arithmetical progressions, permutations and combinations, square, cubes, rules of three, five, seven, etc.
Algebra: Algebraic expressions and their operations, Quadratic and simultaneous equations, etc.
Geometry: Plane and solid geometry, shadow, etc.
Business transactions: Interest calculations, shared investments, prices of different jewels, amount of metals in mixtures, carpentry and land costs, grocery expenses, etc.

The entire book is written in a poetic from. The capability of Bhaskaracharya as a littérateur is evident from the way he puts down the verses (shlokas) in the book. He has made profuse use of alankaras (figures of speech) like pun, alliteration, metaphors and similes in his writing. There are indications that he also had a good sense of humour. For example, in one of his problems in Lilavati a rich man gives so many fractions of coins for many days to the beggar that the total is just one kawadee.

There are about 270 verses in Lilavati. Only a few of them provide guidelines white a majority are used to raise questions. The book begins with an invocation addressed to Lord Ganesha seeking his blessings. I would like to quote it in Sanskrit with English translation by H T Colebrook.

प्रीतिभक्तजनस्य यो जनयते विघ्नं विनिघ्नन स्मृतः ।
तं वृंदारकवृंद्वंदितपदं नत्वा मतंगाननम ॥
पाटीसद्गणितस्य वच्मि चतुरप्रीतिप्रदां प्रस्फुटाम ।
संक्षिप्ताक्षरकोमलामलपदैर्लालित्यलीलवतीम ॥

> *Having bowed to the deity, whose head is like an elephant; whose feet are adorned by gods; who, when called to mind, relieves his votaries from embarrassment; and bestows happiness on his worshippers; I propound this easy process of computation, delightful by its elegance, perspicuous with words concise, soft and correct, and pleasing to the learned.*

There is a famous legend associated with the book Lilavati. It is said that Lilavati was the name of Bhaskaracharya's daughter. He was not only a mathematician but also an astrologer. Looking at the horoscope of Lilavati he realised that her married life could be problematic which could be avoided by marrying her at a particular *muhurta*. To ensure that the marriage takes place on that particular *muhurta* he prepared a *ghatika patra* and left it in water container. It was so adjusted that the marriage ceremony was to begin as the *patra* dipped under water. Impatient and curious Lilavati went to see how far it dipped. While she was standing close to the container a small bid from her ornament fell down and blocked the hole of the container. As a result, the water flow slowed down and the *ghatika patra* dipped later than it should have in normal circumstances. Thus, the marriage ceremony took place later than its planned time.

As the destiny had decided Lilavati became widow and came back to her father after a few years of her marriage. We know that remarriage was not permitted in those days. Hence, widowed Lilavati would remain sad and silent. In order to keep her engaged Bhaskaracharya used to give her interesting problems. As Lilavati was quite intelligent she could solve them quickly. It is stated that the discussion that took place between father and daughter is presented in the form if a book entitled Lilavati.

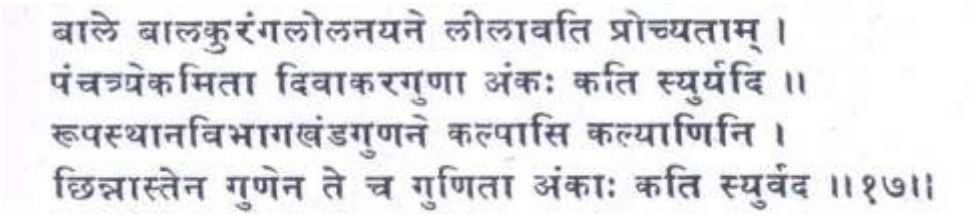
Research into the genesis of Lilavati does not support the above legend. Nonetheless, the picture depicting the interaction between an old man and a young lady is seen at various locations especially on the cover pages of the book Lilavati. It first appeared on the cover page of the Persian translation of Lilavati done by Shri Faizi in 1587 who worked in the *darbar* of king Akbar, the great. Since then the story has been spreading in all parts of the country and is accepted as a fact.

Pedagogical Characteristics

Lilavati is a master piece as far as pedagogical guidelines are concerned. It advocated many useful pedagogical aspects that help the teaching of mathematics. Looking at Lilavati through this angle as a practising mathematics educator I see following pedagogic characteristics of Lilavati.

Gender equality

Whether the legend stated above is true or not one thing is clear that Bhaskaracharya advocated gender equality through his writings. At many places in Lilavati he refers to readers as bale, sakhe, charming girl, etc. One must keep in mind that those were the days when girls were discouraged from formal schooling. Even when the education has been opened to female members they opted for softer subjects and avoided hard disciplines like mathematics. It was certainly revolutionary on the part of Bhaskaracharya to address mathematical problems to a female member through his writings. An example in this case would support what I am saying.


बाले बालकुरंगलोलनयने लीलावति प्रोच्यताम् ।
पंचत्र्येकमिता दिवाकरगुणा अंकः कति स्युर्यदि ॥
रूपस्थानविभागखंडगुणने कल्पासि कल्याणिनि ।
छिन्नास्तेन गुणेन ते च गुणिता अंकाः कति स्युर्वद ॥१७॥

A beautiful and dear Lilavati, whose eyes are like fawn's, tell me the numbers resulting from one hundred thirty five taken into twelve, if thou be skilled in multiplication by whole or parts whether by subdivision of form or separation of digits.

Building prerequisites

Mathematics is a hierarchical subject. All that is learned earlier is essential to progress further in mathematics. If a student does not possess requisite knowledge or skills he/she would not be able to handle new situation. From the sequence of examples in Lilavati it is clear that Bhaskaracharya makes deliberate efforts to build essential prerequisites. The examples given under a specific topic are so graded that students develop all the necessary prerequisites knowledge and skills before reaching a major problem in the section. This helps them build confidence and deal with the main problems without any difficulty.

Motivation

Motivation is considered to be an essential component of learning. Unless the person is motivated the learning would not take place even if the teacher has completed teaching. Importance is, therefore, given in school teaching to motivate the learner even before a task is given. Bhaskaracharya seems to be aware of this pedagogical fact. Hence, he tries to create interest among the readers through a variety of means. In some cases he presents the problem through a story. In other cases he creates interesting situations. A problem involving snake and peacock can be considered as an example in this category.

अस्ति स्तंभतले बिलं तदुपरि क्रीडाशिखंडी स्थितः|

स्तम्भेहस्तनवोच्छिते त्रिगुणितस्तंभप्रमाणांतरे ||

दृष्टवाहिं बिलमाव्रजंतमपतत तिर्यक स तस्योपरि |

क्षिप्रं ब्रूहितयोर्बिलात्कतिमितैः साम्येन गत्योर्युतिः ||

A snake hole is at the foot of a pillar, nine cubits high, and a peacock is perched on its summit. Seeing the snake at a distance of thrice the pillar gliding towards the hole, he pounces obliquely upon him. Say quickly how many cubits from the snake's hole they meet, both proceedings an equal distance.

Rapport building

Most of the times the book on mathematics prescribed for formal study is written in a dull and dry style. The entire book Lilavati is, however, written in first person active voice. The author addresses the reader with loving words like mitra, bale, sakhe, etc. At many places he addresses the readers as mathematician and instigates them to undertake problem solving.

Open endedness is the hallmark of Lilavati. Bhaskaracharya has not solved any problem in his book. He suggests multiple ways of dealing with a problem and allows the reader to choose the

method of his or her choice. At some places he provides necessary hints but does not force the reader to follow a specific method. Here is an example from Lilavati as a part of kuttak method. I am directly presenting its English translaltion.

> *Say quickly, mathematician, what is that multiplier, by which two hundred and twenty-one being multiplied, and sixty-five added to the product, the sum divided by a hundred and ninety-five becomes exhausted."*

Content relevance

Studies show that students dislike mathematics as they find it irrelevant to their daily lives. Realising this fact Bhaskaracharya attempts to pose problems of daily relevance like income, area, stacking of bricks, etc. The problems are composed taking animate as well as inanimate objects found around. Thus snakes, bees, swans, elephants, etc. are referred to in the description. Apart from showing the relevance these problems create interest among the students for problem solving. Here is one such example from Lilavati.

यातं हंसकुलस्य मूलदशकं मेघागमे मानसं ।
प्रोड्डीय स्थलपद्मिनीवनमगदष्टांशको भस्तटात ॥
बालेबाल्मृणालशालिनि जले केलिक्रियालालसम ।
दृष्टंहंसयुगत्रयं च सकलां यूथस्य संख्या वद ॥

> *Out of the swans in a certain lake, ten times the square root of their number went away to Manasa Sarovara when rains started, and one eighth the number went away to the forest Sthala Padmini. Three pairs of swans remained in the tank, engaged in water sports. What is the total number of swans?*

Field Experiences

In order to celebrate the 900th birth anniversary of Bhaskaracharya the Vidya Prasarak Mandal (VPM), Thane has decided to organise an international conference on the life and work of Bhaskaracharya. In addition, workshops were planned for students and teachers based on Lilavati. Starting from January about 50 workshops have so far been conducted in different schools and colleges in the states of Maharashtra, Madhya Pradesh and Andhra Pradesh. These workshops are conducted in two parts. The first part is devoted to familiarising the participants with the rich tradition of Mathematics in India. Bhaskaracharya is presented as a member of this tradition. Some information about his life and work is outlined. The second part of the workshop is used to deal with selected problems from Lilavati. These problems are chosen taking into account the educational background of the participants.

The experiences of conducting these workshops have been quite positive. Students, at different levels, find the task of solving problems from Lilavati very interesting. It is notable that the participants engage themselves in the task for almost three hours. On many occasions they

come forward to solve problems on the blackboard. Due to educational practice that is followed in the school they do not appreciate the open ended approach advocated by Bhaskaracharya to begin with. Soon however, they start realising the importance of this method and come out with novel method of dealing with given problem. Let me share an experience that I had while solving the snake and peacock problem mentioned above. Standard method that is followed in dealing with the problem is to apply Pythagoras theorem and find out the value of requisite distance. One boy however, came out 12 as the answer quickly. On enquiry he revealed that he tried to look for a triad having number 9 in it. Another boy went on adding square numbers to 81 (square of 9) and checked whether the answer is a square number. Through this method he also came out with an answer 12.

Conclusion and implications

It is clear that the pedagogical Characteristics of Lilavati are relevant even today. Efforts must be made to practice them in day to day teaching. I believe that the methods adopted by Bhaskaracharya would work both for first generation learners and traditional learners. It would, I am sure, enable us to make Indian citizens mathematically literate. We should be proud that we have an age old tradition of teaching mathematics effectively. It is high time that we make use of this traditional knowledge to improve teaching of mathematics in Indian schools and colleges. Constructivistic approach, Situational Learning and Experiential learning are the buzz words of present day pedagogy. Without saying it explicitly, Bhaskaracharya had adopted all of these pedagogic techniques in his writings. The very meaning of his name is Bhaskar the teacher (Acharya in Sanskrit means teacher). I once again salute this great teacher of India. Thank you for your patient listening.

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